Assessment of Adherence to the Condition of Proportionality in User Equilibrium Traffic Assignments with Uniquely Determined Route Flows

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ABSTRACT

The standard formulation of static deterministic user equilibrium (UE) traffic assignment problem based on the criterion of Wardrop provides a unique solution in terms of link flows; however, route flows are not determined uniquely. Analyses based on an arbitrary choice among the infinite number of possible route flow solutions could cause inconsistencies or even controversies in applications. In 2010, a computationally efficient algorithm called Traffic Assignment by Paired Alternative Segments (TAPAS) was successfully implemented to identify UE route flows uniquely. So far, no effort has been made to assess adherence to the condition of proportionality in UE traffic assignment with uniquely determined route flows. In this paper, TAPAS was solved to obtain proportional UE route flows for the Chicago regional network in the closest proximity to uniqueness of the solution. Various assessments of adherence to proportionality are performed for a selected pair of alternative segments. The results show that route flows determined by TAPAS correspond closely to exact proportionality. Only minor differences occur between computed and exactly proportional UE route flows. Systematic characteristics of the plots for the two alternative segments show that TAPAS behaves properly according to the condition of proportionality. Insights from these empirical results may help transportation planning professionals to be aware of the magnitude of differences in UE route flows based on proportionality and to be able to differentiate uniqueness from non-uniqueness of route flows in UE traffic assignment. The results may also be useful to software developers in seeking improved adherence to proportionality of route flow solutions.

Keywords: traffic assignment, the condition of proportionality, pairs of alternative segments

INTRODUCTION

The standard formulation of the static deterministic user equilibrium (UE) traffic assignment problem based on the criterion of Wardrop provides a unique solution in terms of link flows; however, route flows are not determined uniquely. Analyses based on an arbitrary choice among the infinite number of possible UE route flow solutions could cause inconsistencies or even controversies in applications. In 1999, a behaviorally justifiable condition of proportionality on paired alternative segments was first proposed by Bar-Gera to determine route flows consistently (*1*). Unfortunately, proportionality is only a necessary, but not a sufficient condition to identify route flows uniquely. In 2006, Bar-Gera proposed the condition of route set consistency to obtain a set of routes that is likely to be similar to the exact set of UE routes and showed that this condition suffices for any solution that aims to satisfy uniqueness of route flows (*2*). In 2010, a computationally efficient UE solution algorithm called Traffic Assignment by Paired Alternative Segments (TAPAS) was successfully implemented to identify UE route flow solutions that concurrently satisfy the conditions of proportionality and route set consistency (*3*).

Due to its ability to satisfy the condition of proportionality within used routes, TAPAS was widely applied as reference solutions in evaluating consistency, or adherence to the condition of proportionality, of various UE route flow solutions *(4, 5, 6).* Results generated with TAPAS available thus far have only considered relative gap and no effort has been made to assess adherence to the condition of proportionality in UE traffic assignment with uniquely determined route flows. In this paper, proportionality is precisely enforced under the most reasonable conditions of UE and route set consistency to obtain proportional route flows in the closest proximity to uniqueness of the solution. Various assessments of adherence to the condition of proportionality are performed for a selected pair of alternative segments over three single-class congestion scenarios for the Chicago regional network. Selected results are presented in a way that transportation planning professionals may find helpful in understanding underlying solution characteristics of unique UE route flows and in differentiating uniqueness from non-uniqueness of route flows in UE traffic assignment.

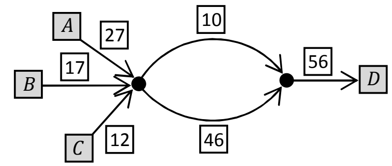
In the remainder of this paper, the issue of non-uniqueness of UE route flow solutions as well as consistent treatments for uniqueness of the solutions are offered, followed by brief descriptions of the model and overall algorithm solutions. Principal findings regarding proportionality assessments, effects of proportionality on individual link flows, and differences between computed and exactly proportional UE route flows are then presented. Finally, conclusions, usefulness, and future research directions complete the paper.

CONSISTENT CHOICES OF UE ROUTE FLOWS

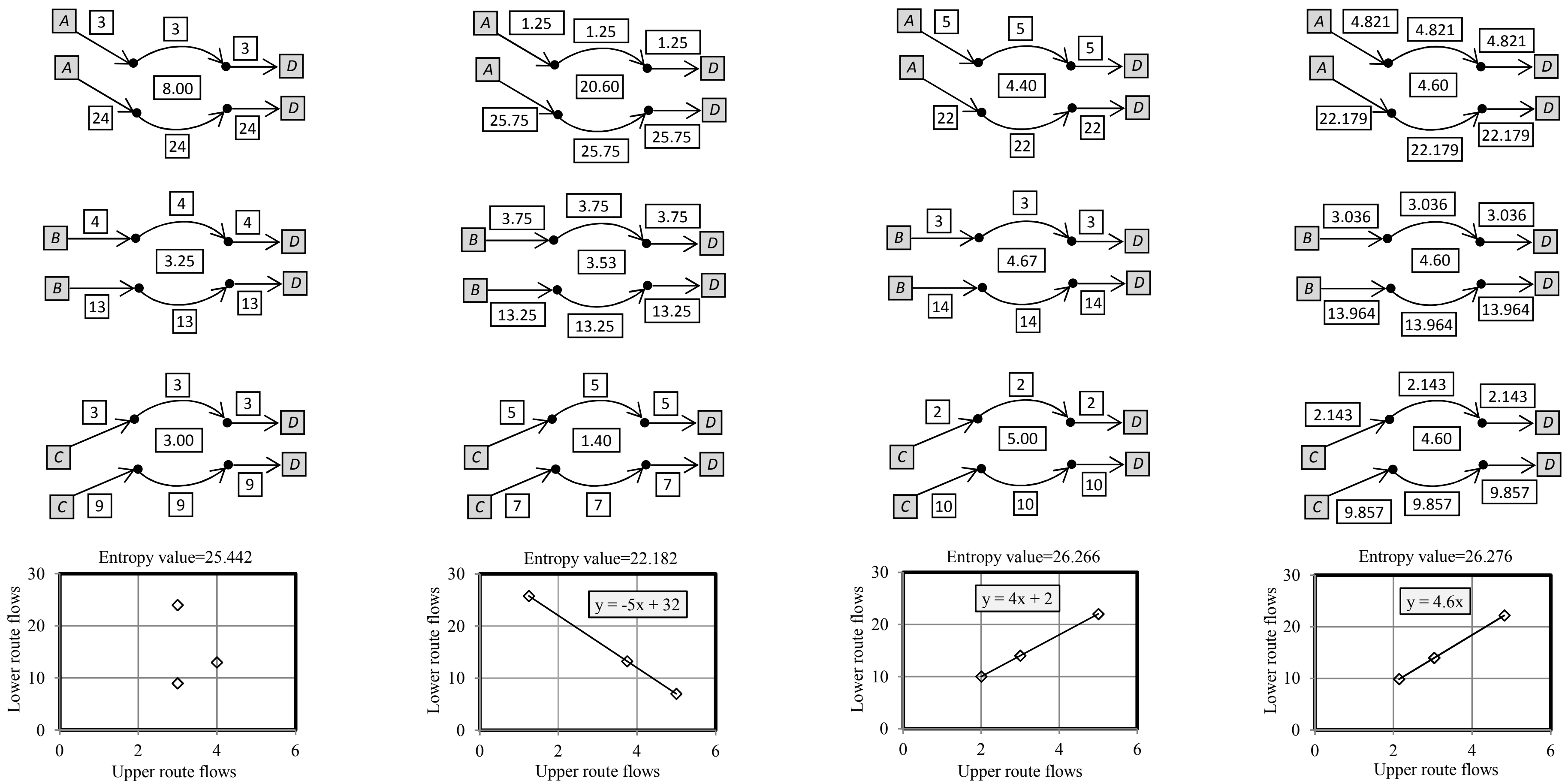
This section aims to address the issue of non-uniqueness of route flows under the UE condition as well as to introduce two equivalent approaches in choosing route flows uniquely. Figure 1(a) shows a UE link flow solution for a simple road network, which consists of three O-D pairs connecting three origins to one destination. Assume that link flows shown beside each link in Figure 1(a) represent a perfect UE solution. In principle, the number of possible UE route flow solutions that correspond to the same link flows of any road network can be infinite; four solutions that correspond to the same link flows are shown in Figures 1(b)-1(e) by O-D pair. Each O-D pair is connected by two alternative routes. Assume further that all equilibrium routes shown in each solution represent an exact set of UE routes. Route flows are denoted along the links that comprise the route. Below each route flow solution is a scatter plot of vehicle flows along the routes; each point represents flows for one O-D pair passing through the upper alternative route (x-axis) and the lower alternative route (y-axis) respectively. The trend line and linear regression equation are shown on each scatter plot.

The question of how many vehicles from each origin use each of the two alternative segments arises naturally. Clearly, the results from the four route flow solutions are different. For example, flows on the upper alternative segment from origin A are substantially different, varying from 1.25 in solution 1(c) to 5 in solution 1(d). Without any specific mathematical criterion or behavioral assumption to support decisions, the choice among the infinite number of possible UE route flow solutions is arbitrary and may cause inconsistencies or even controversies in applications. The early development for consistent choices of UE route flows is based on the criterion of entropy maximization *(7, 8)*. The entropy function gives the probability of route choices made by individual travelers within a specific route flow pattern. Since entropy presumes an equal probability of occurrences for each route choice, any pattern that occurs most frequently is regarded as the most likely route flows (*8)*. In principle, maximizing entropy subject to the constraints that the total link flows are UE leads to the identification of the most likely route flow pattern. Solution 1(e) is the one and only one result that maximizes entropy. Entropy values, which are provided on top of each scatter plot, show that route flows in solution 1(c) are farthest away from being the most likely route flows, followed by those in solution 1(b) and 1(d) respectively.

An alternative approach to choosing route flow solutions consistently is based on route choice behavior of individual travelers. This approach implicitly assumes that all individual travelers, no matter where they are from or going, are rational in their behavior; therefore, if facing choices between the same two alternative route segments, they should distribute themselves over the two alternative route segments in the same proportions. This assumption is formally known as the condition of proportionality, which was first introduced by Bar-Gera and Boyce in 1999 as an intuitive behavioral interpretation for necessary conditions that characterize entropy maximizing route flows *(1)*. To help examine individual route choice behavior in each route flow solution, the ratio of travelers traversing the lower to upper alternative route segment, namely the slope of a straight line that passes through the origin of a scatter plot, is denoted in the middle of each solution subfigure. Inconsistent ratios of travelers across three origins in solutions 1(b)-1(d) suggest that the assumption of proportionality is violated in all three solutions in that not all individual travelers behave the same across origins; they only behave similarly



(a) A simple road network with a perfect UE solution on total link flows



(b) An arbitrarily assigned UE route flow solution

(c) A disproportionately assigned UE route flow solution

(d) A disproportionately assigned UE route flow solution

(e) A proportionately assigned UE route flow solution

Note that entropy is defined by $\mathrm{E}(\boldsymbol{f}) = -\sum_r \sum_s \sum_k f_k^{rs} \ln \frac{f_k^{rs}}{q_{rs}}$;
where f_k^{rs} is the flow on route k connecting O-D pair rs and q_{rs} is total O-D demand flow between origin r and destination s.

FIGURE 1 Consistent choices of UE route flows.

within the same origin but differently across origins. Without any supportive rationale, solutions 1(b)-1(d) should not be considered for use in any meaningful analysis. Only all individual travelers in solution 1(e) behave intuitively and distribute themselves proportionately over the two alternative route segments across all origins; that is, within-each-origin proportionality is the same as between-origin proportions in a fashion that the ratios of travelers are identical across all three origins and equal to the slope of the regression line. Basically, the two alternative segments of the equal cost routes are referred as a pair of alternative segments or "a PAS". By definition, every PAS consists of one diverge node, one merge node, and two equal cost segments; each segment is defined by a distinct sequence of one or more directed links and an identical set of relevant origins.

MODEL DESCRIPTION AND OVERALL ALGORITHM SOLUTIONS

TAPAS, which is used to prepare the results presented in this paper, is an iterative traffic assignment algorithm designed to identify UE route flow solutions that satisfy conditions of proportionality and route set consistency *(3)*. TAPAS was utilized to distribute O-D flows from three single-class trip matrices to the Chicago regional network, which consists of 1,790 zones, 12,982 nodes, and 39,018 links. Each trip matrix was constructed according to a mode-origin-destination-trip distribution model with doubly-constrained logit form, $d_{mpq} = A_p B_q \exp(-\mu \cdot c_{mpq})$, where (A_p, B_q) are balancing factors, μ is a cost sensitivity (CS) parameter, and c_{mpq} is mode-origin-destination generalized travel cost. Three trip matrices only differ by the value of CS parameter: 0.20, 0.10, and 0.05. The largest value has the highest sensitivity to cost, the lowest generalized travel cost, and the least congestion. The generalized link travel cost is assumed to equal link travel times given by the conventional BPR function. Details regarding the three trip matrices are available in Bar-Gera and Boyce (*9*).

To assure that proportionality is precisely converged under the most reasonable conditions of UE and route set consistency, TAPAS was terminated at 200 iterations for the 0.20 and 0.10 solutions and 1,500 iterations for the 0.05 solution. Additional refinements did not show to improve convergence of the solutions. In this paper, TAPAS achieved a sub-consistency ratio of 3.02E-4 for the 0.20 and 0.10 solutions and a super-consistency ratio of 81 for the 0.05 solution. The first two solutions fail to reach super-consistency because they require the precise level of convergence that is well beyond the precision limits of present computer technology. Unlike past studies, all solutions in this paper were computed to the maximum relative gap of 4.9E-16 and the maximum flow deviation from proportionality of 8.45E-9 *(10)*. Proportional route flows in this paper are, therefore, considered as the closest proximity to uniquely determined UE route flows. Formal definitions of three convergence measures are found in Bar-Gera (*3*).

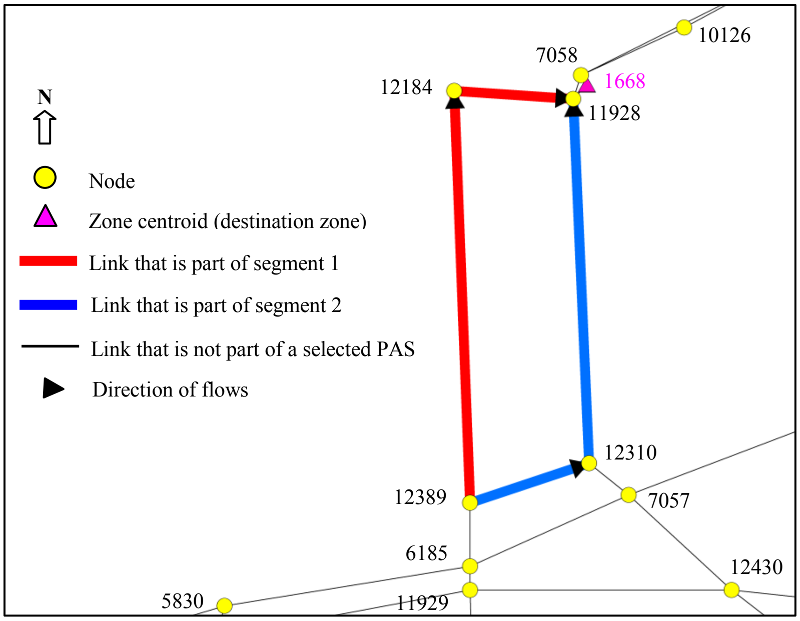
PROPORTIONALITY ASSESSMENTS

In Chicago regional network, the number of PASs identified in forming uniquely determined UE routes for the 0.20, 0.10, and 0.05 solutions are 5,617, 11,702, and 22,500 respectively *(10)*. Comprehensive analysis of individual PASs does not seem to be a reasonable basis for proportionality assessing. In this paper, only one PAS with the maximum number of relevant origins is selected from each solution to display the principal findings. Although this approach is not statistically representative, there is a good reason to believe that it does provide enough evidence to see what might occur with other PASs. A basis for this selection is that PASs with fewer numbers of relevant origins will adhere to proportionality in the same way as that with maximum number of relevant origins. Figure 2 shows a map of a selected PAS for three solutions. Past studies experimenting on the same road network with the same values of CS showed that a simple PAS formed by four links occurs most frequently *(10)*. The selected PAS of four links is typical of all PASs, assuring that findings and conclusions obtained from this paper are representative. In the following, adherences to the condition of proportionality are assessed through selected paired segments analyses, which are performed at two levels: by aggregate O-D pairs and by disaggregate O-D pairs.

Selected Paired Segments Analysis at Aggregate Levels

Key attributes for the three solutions summarized beneath a map of the selected PAS in Figure 2 are helpful in interpreting analyses in subsequent sections. Information regarding the number of relevant origins and destinations suggests that flows traversing the two segments come from almost everywhere in the region and the destination zone is the same for all origins. It seems surprising that a very small PAS is used by almost all origins. However, this is most favorable for TAPAS. In principle, PASs with shorter total link lengths and fewer links are more computationally desirable for they are likely to be relevant to more origins, and a shift of flows between pairs of routes can be computed faster *(2).* Recall that one O-D pair may consist of one or more disaggregate O-D pairs; each disaggregate O-D pair is defined by the two routes that only distinguish at alternative segments. For an O-D pair with more than one disaggregate O-D pairs, route segments taken from the origin to the diverge node and/or from the merge node to the destination for one disaggregate O-D pair must be physically different from others. Information regarding the number of O-D pairs and unique UE routes suggests that most O-D pairs have more than one pair of UE routes or more than one disaggregate O-D pair. For example, there are 7,926 different UE routes connecting 1,781 O-D pairs or equivalently 3,963 disaggregate O-D pairs in the 0.20 solution.

In each solution, between-origin proportionality is observed on each of the two equal cost segments. Comparisons between solutions reveal that congestion and proportionality are not necessarily related. In fact, the effects of congestion on proportionality are PAS-specific, meaning that among the solutions the proportions of flows over the two segments for a given PAS respond to congestion differently. For example, the proportions of flows on one segment



Cost sensitivity	CS-0.20		CS-0.10		CS-0.05	
Segment	1	2	1	2	1	2
Total number of links in a segment	2	2	2	2	2	2
Total link lengths in a segment (mi)	1.49	1.07	1.49	1.07	1.49	1.07
Total flows on a segment (vph)	284.249	543.627	255.189	542.283	108.502	585.433
Total costs on a segment (auto in-vehicle min)	2.676	2.676	2.673	2.673	2.667	2.667
Exact proportion of flows on a segment	0.343	0.657	0.320	0.680	0.156	0.844
Total origins relevant to a segment	1,781	1,781	1,781	1,781	1,775	1,775
Total destinations relevant to a segment	1	1	1	1	1	1
Total O-D pairs relevant to a segment	1,781	1,781	1,781	1,781	1,775	1,775
Total disaggregate O-D pairs relevant to a segment	3,963	3,963	10,331	10,331	21,471	21,471
Total unique UE routes relevant to a segment	3,963	3,963	10,331	10,331	21,471	21,471
Total number of links relevant to a segment	8,207	8,207	8,495	8,495	8,916	8,916

FIGURE 2 Selected paired segments analysis at aggregate levels.

could increase or decrease as congestion grows. For this particular selected PAS, the proportions of flows over the two segments are substantially influenced by congestion levels on the network. The proportion of flows in each solution is higher on segment 2 than segment 1. As seen, the difference between segment proportions in the 0.20 and 0.10 solutions is about half that of the 0.05 solutions. The decreases in total flows over the two segments from the 0.20 to 0.10 to 0.05 solution pertain to two intertwined factors: a decrease in CS values, and an increase in link and route costs resulting from increased traffic on the network. Since the segment proportions shown in Figure 2 are computed from aggregations of flows on all routes traversing each of the two alternative segments, they are regarded as the exact proportionality with which every disaggregate O-D pair must agree.

Selected Paired Segments Analysis at Disaggregate Levels

A way to assess adherence to proportionality of (disaggregate) O-D pairs is to make scatter plots of the data. Each data point shown in Figure 3(a) represents one O-D pair plotted on the linear scale. The top row shows plots of total O-D flows (x-axis) against O-D flows for routes traversing segment 1 (y-axis) for three solutions, arranged by decreasing values of CS from the left to the right. The middle row shows the same plots as with the first row, with the y-axis represents O-D flows for routes traversing segment 2. Plots in the bottom row shows flows of a particular O-D pair split between the two segments. Above each trend line is a linear regression equation in conjunction with a statistical measure of R^2 to assist assessments of adherence to proportionality. To enhance interpretation of the plots, the total number of O-D pairs are also shown above each plot. As seen in each plot, all the points fall along one straight line through the origin (0,0), indicating that for this pair of segments the same proportion of flows is perfectly applied to all O-D pairs. The extremely high degree of adherence to proportionality indicated by $R^2 = 1$ is not unexpected for TAPAS assignments in which the solutions are highly converged and the condition of proportionality is precisely enforced for every pair of alternative segments on the network. The observed differences in the proportions from solution to solution result from sensitivity to travel cost and congestion on the network, and not from imprecise solutions or arbitrary choices of used routes over a selected pair of alternative segments. The slopes of the trend lines on the first two rows correspond precisely to the exact proportions, indicating that every disaggregate O-D pair agrees with the exact proportion and that TAPAS works perfectly in assigning the O-D flows to the two segments in the same proportion. The three different slopes of the trend lines on the bottom row reflect three different ratios between the flows on segment 1 and 2. Assuming that each slope is represented by a constant m, the plots suggest that the vertical coordinate of each point in the middle row is m times those in the top rows. To put it another way, for every disaggregate O-D pair the flow on the route traversing segment 2 is m times that of segment 1. Without the imposition of proportionality, the ratio of the flows on segment 1 to the flows on segment 2 in each solution could be rather arbitrary, resulting from non-uniqueness of route flows.

One may be interested to know whether the proportions on segment 1 and 2 are actually the same for every O-D pair. Plots shown in Figure 3(a) do not permit visual comparisons for every O-D pair adheres to proportionality so perfectly that the differences in the proportions among O-D pairs are not visible in any plot. To amplify the differences, plots in Figure 3(a) are slightly modified as shown in Figure 3(b). The similar layouts of the plots are maintained for ease of comparison. For the plots of the first two rows, the O-D flows on the x-axis are plotted on the log scale to spread out the points by different orders of magnitude, ranging from -14 to 4. As an example, an x-axis value of -14 corresponds to an O-D flow equal to 1E-14 vph. To obtain the proportions of flow on segment 1 and 2, each value on the y-axis is divided by its total O-D flows. Since segment proportions shown in Figure 3(b) are computed from the flows of only one disaggregate pair of routes that traverse each segment, the y-axis value of each data point is

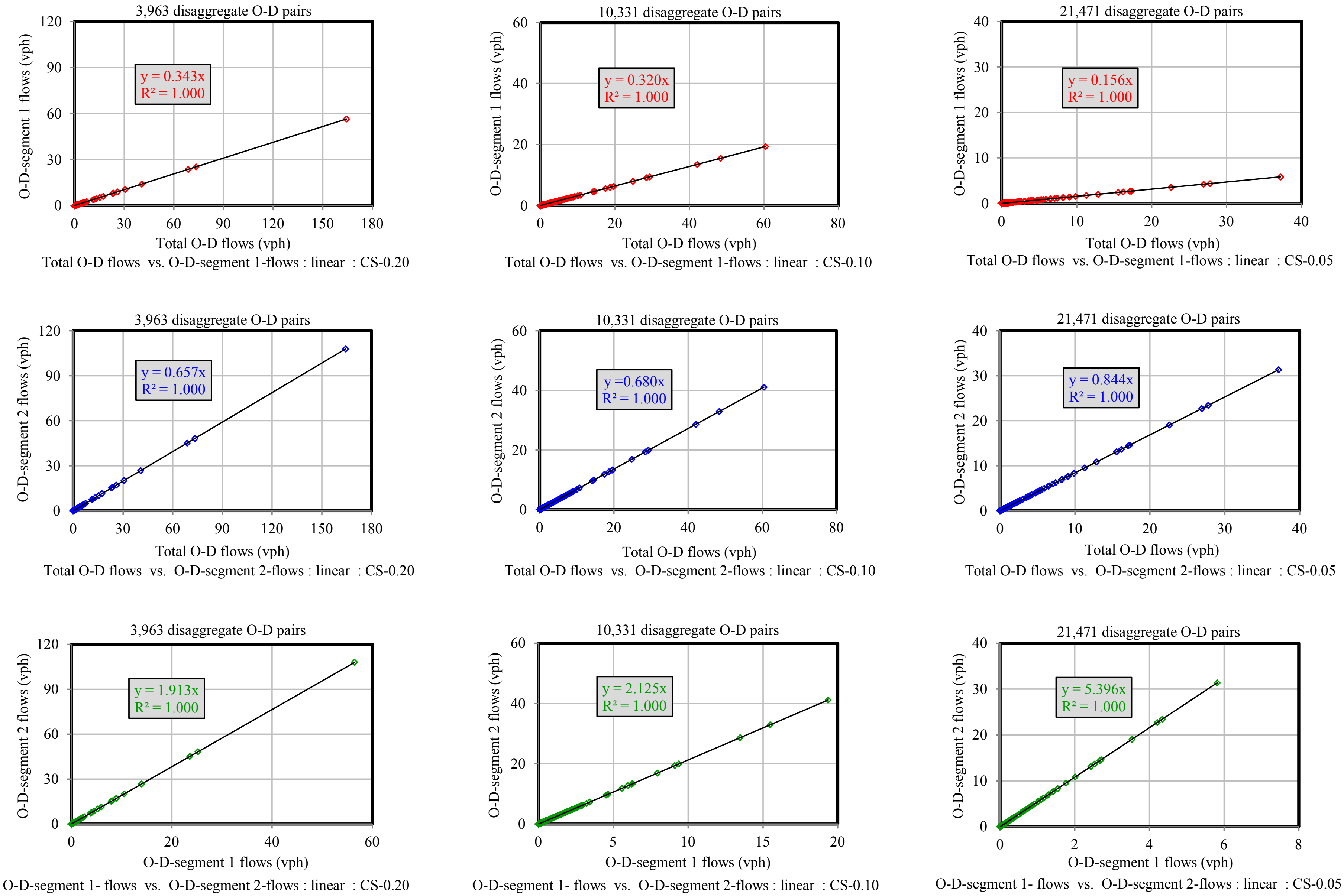
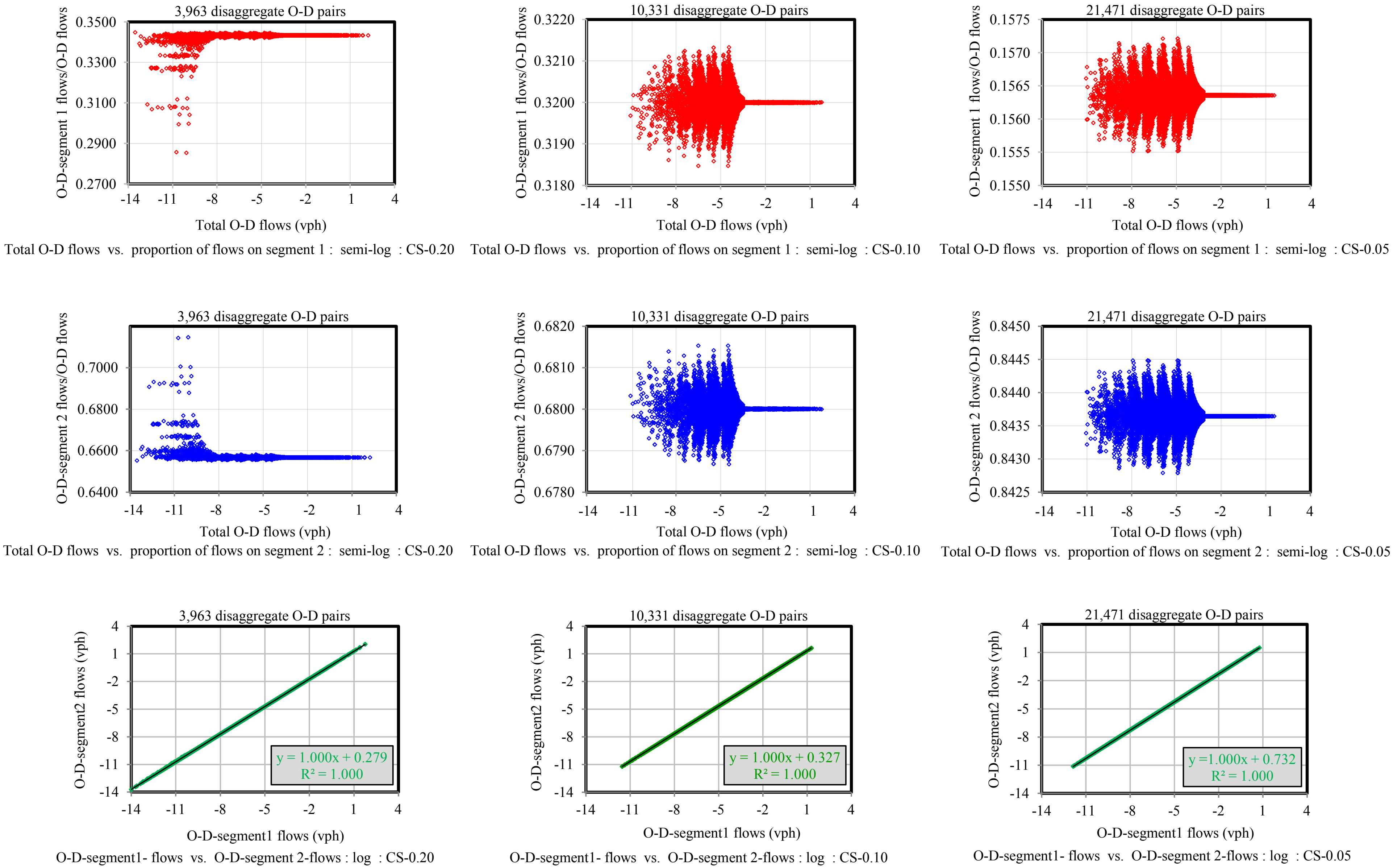


FIGURE 3(a) Selected paired segments analysis at disaggregate levels:- assessed by O-D-segment flows.



Total O-D flows vs. proportion of flows on segment 1 : semi-log : CS-0.20

Total O-D flows vs. proportion of flows on segment 1 : semi-log : CS-0.10

Total O-D flows vs. proportion of flows on segment 1 : semi-log : CS-0.05

Total O-D flows vs. proportion of flows on segment 2 : semi-log : CS-0.20

Total O-D flows vs. proportion of flows on segment 2 : semi-log : CS-0.10

Total O-D flows vs. proportion of flows on segment 2 : semi-log : CS-0.05

O-D-segment1- flows vs. O-D-segment 2-flows : log : CS-0.20

O-D-segment1- flows vs. O-D-segment 2-flows : log : CS-0.10

O-D-segment1- flows vs. O-D-segment 2-flows : log : CS-0.05

FIGURE 3(b) Selected paired segments analysis at disaggregate levels:- assessed by proportions of flows.

regarded as the computed proportionality. Since the proportions must lie between 0 and 1, a linear scale is suitable for used to display the magnitude of the differences in the proportions among O-D pairs

The semi-log plots on the first two rows exhibit notable patterns of reverse symmetry in which a plot in the top row is a mirror image of that in the middle row. These reverse symmetry patterns result from the complementary effects of flow shift operations made between any pair of routes. Shifting the same flows from a higher cost route to a lower cost route does not result in any change in the total O-D flow. As a result, the proportions of flow for routes traversing segment 1 and 2 are complementary in the sense that two proportions, when added together, will equal unity. The presence of a reverse symmetry pattern ensures that TAPAS behaves properly and the properties of the PASs are as expected. As can be seen, the variations in the proportions of flows only occur with regard to O-D pairs with flows less than 1E-3 vph, whereas O-D pairs with flows greater than 1E-3 vph seem to correspond best to the exact proportions as indicated by the horizontal alignment of the data points. Since these variations are only shown to be in the very narrow range of very small values, they are most likely caused by rounding errors, which could not be eliminated perfectly. These extremely small variations can be largely attributed to the high precision of TAPAS in computing the proportional UE route flow solution. The very small O-D flows are a characteristic of how the trip matrices were computed.

The plots in the bottom rows of Figure 3(b) show exactly the same points as those in Figure 3(a), but they are shown in log scale to allow a closer look at O-D pairs with small segment flows. The values shown on the axes are the orders of magnitude of O-D flows traversing the two segments. As indicated by the coefficient of linear regression equations and corresponding values of R^2, all the points in each solution perfectly fit the 45-degree line, but they belong to different intercepts on the y-axis. The y-intercept of the log plots corresponds to the slope of the linear plots. Positive signs of the y-intercepts simply indicate there are more O-D flows on route traversing segment 2 than segment 1. An agreement of all data points with the 45-degree line ensures that O-D flows are split proportionately between the two alternative segments.

EFFECTS OF PROPORTIONALITY ON INDIVIDUAL LINK FLOWS

Proportionality affects not only the flows on routes traversing the two segments, but also the flows on individual links of routes traversing segments. Figure 4(a) shows scatter plots for flows on routes traversing a selected segment aggregated by links. Each data point represents one link. For the plots in the first two rows, the x-axis represents the link number; whereas the y-axis represents the total flows on all routes traversing both that link and the segment on the log scale. For the plots in the bottom row, the x and y axes simply correspond to the vertical axes of the top and middle rows but plotted on the linear scale. The slope at each point corresponds to the ratio of total flows on a link for routes traversing segment 2 to total flows on that link for routes traversing segment 1. Parameters of linear regression are estimated and shown in the body of the plots. Note that total number of links shown on the top of each plot does not include links that are part of the selected PAS nor zone connectors. As compared within the solution, the patterns of total link flows given by the plot in the top row are exactly identical to those in the middle row. Identical total link flow patterns are a direct consequence of proportionality, which can be observed by a single alignment of points in the bottom row. Notice that a linear regression equation in the bottom row of Figure 4(a) is the same as that in Figure 3(a), reaffirming proportionality on link flows. Without the inclusion of proportionality, these link flow patterns could be arbitrarily different. As compared across the solutions, the wider ranges of total link flows found in the 0.20 solution become narrower and are shifted upwards in response to growing congestion in the 0.10 and 0.05 solutions. However, flows on links for routes traversing segment 2 are higher than those traversing segment 1 and the relative total flows on each link are equal to the slope of the line shown in the bottom row.

To allow one to observe spatial distributions of individual link flows over the entire road network for each solution, each data point of the plots in the top and middle row of Figure 4(a) is slightly modified and translated onto a map of the physical road network. In the map, the total flow on a given link is shown as a percentage of segment flows. Displaying all possible values of total link flows relative to segment flows on map is rather cumbersome and not necessary. Therefore, the values shown are categorized into eight different scales, ranging from 0 percent for links that are unused by the routes traversing a selected segment to 100 percent for links that are part of a selected segment. Maps corresponding to each solution are, for ease of comparisons, placed side by side and shown in Figure 4(b). Supplementing each map, two sets of solution attributes are summarized on the upper right of the map: attributes for a selected segment shown on the top half are reiterated to enhance interpretations of proportionality, and attributes for the routes traversing a selected segment shown on the bottom half are summarized to provide fundamental insights into effects of proportionality on route flow solutions. In each solution, the black lines represent a set of PASs that comprises initial route segments taken from any origin to diverge node 12389. Since the initial route segments are commonly used by paired routes, both sets of PASs are common regardless of which selected segments are used. As a result, unique UE route segments taken from any origin to diverge node 12389 are identical. The area marked by a small circle represents the geographical location of the selected PAS. Notice that the two previously excluded links that are part of a selected segment are currently included in the total number of links. As seen in each map, total route flows equal segment flows. Minimum route flows indicate that no route flow over the two selected segments is zero, confirming the fundamental principle of a PAS. Identical link flow patterns may be observed between the two maps. Since proportionality is applied in each solution, the percentage of flows on any specific link relative to flows on segment 1 must be identical to the percentage of flows on the same link relative to flows on segment 2. Without the imposition of proportionality, arbitrary differences in flows on links for routes traversing segment 1 and 2 would be expected. Equal mean travel costs for routes traversing segment 1 and 2 are the results of imposed proportionality as well.

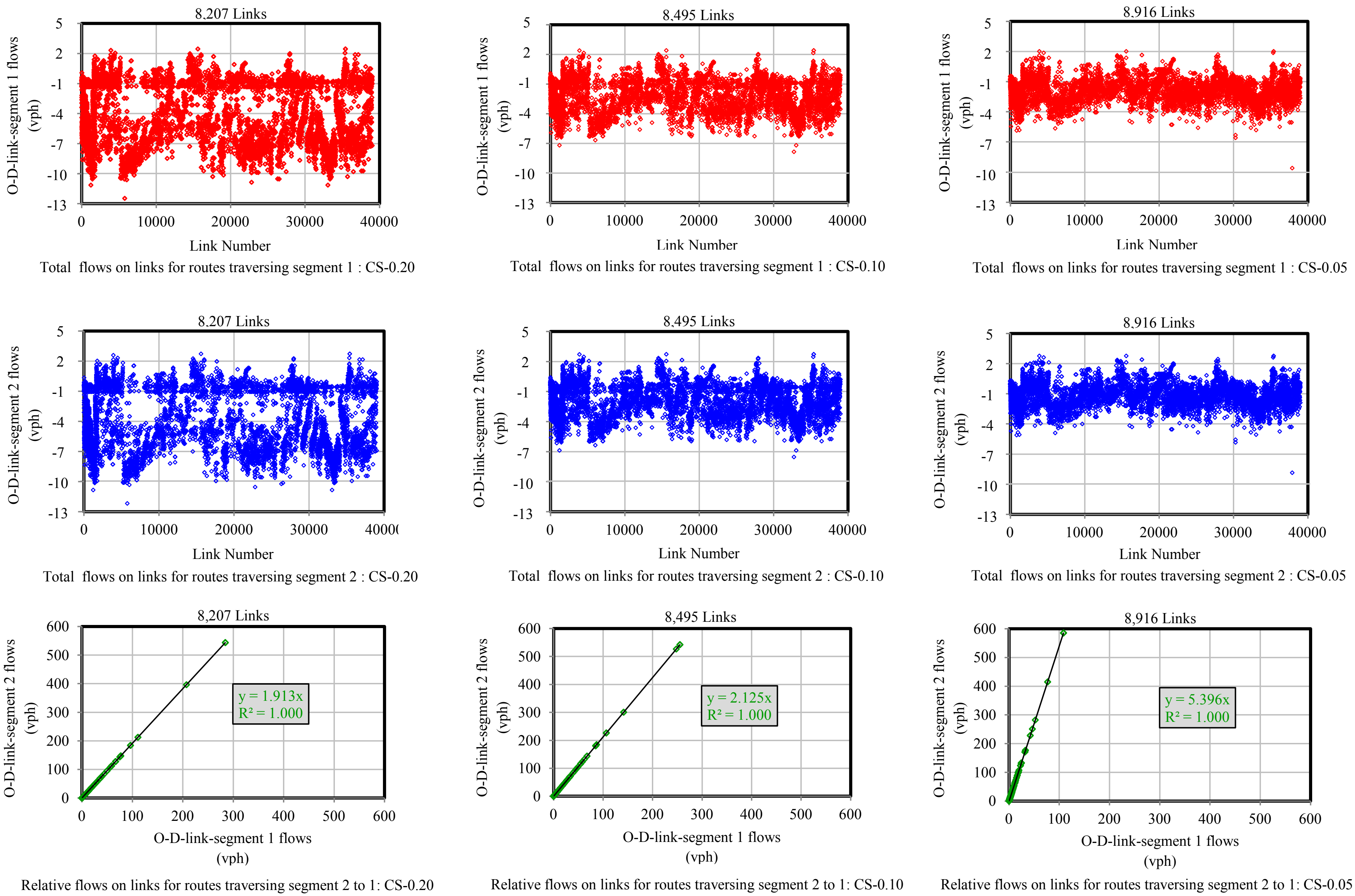


FIGURE 4(a) Plots showing the effects of proportionality on individual link flows for the three solutions.

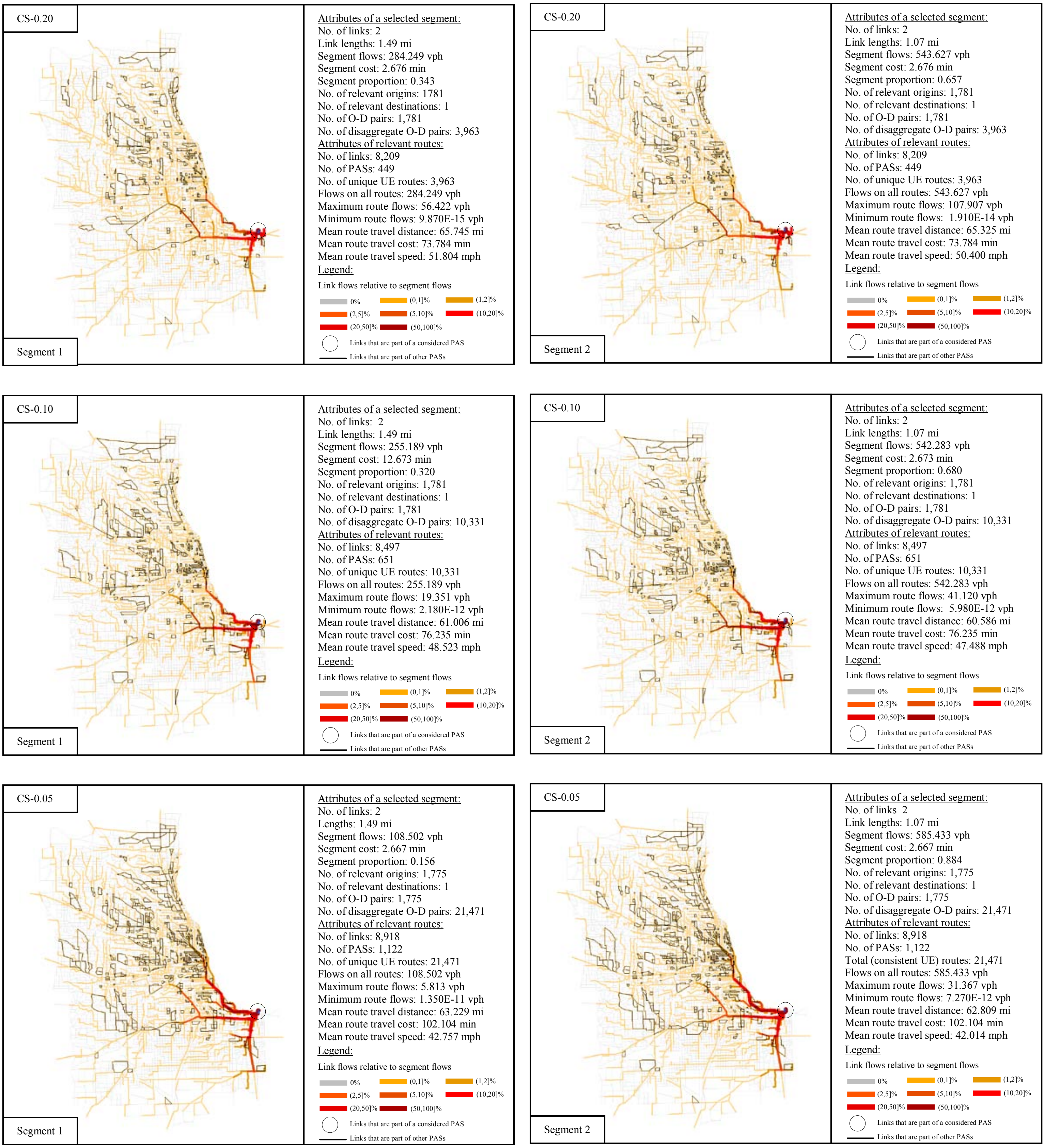


FIGURE 4(b) Illustrations for the effect of proportionality on individual link flows for the three solutions.

By comparing between solutions, one may observe that the number of PASs and the number of uniquely determined UE routes depend to a substantial extent on the congestion levels in the network. Nonetheless, the number of PASs in each solution is relatively small in comparison to the number of uniquely determined UE routes. Substantially higher values of mean route travel costs in the 0.05 solution indicate to what extent the travelers in this solution are insensitive to travel costs, as compared with those in the 0.20 solution, which is considered to be somewhat realistic. For uncongested networks higher route travel costs primarily result from longer route travel distances; however, these characteristics are not necessarily valid under congested networks. Therefore, the levels of congestion between the solutions can be alternatively measured by mean route travel speed, which unifies both route travel distances and costs.

In each solution, there are three major corridors in which flows on links are distinct from each other. Each corridor in each solution has a fairly close correspondence to its geographical location. However, the corridors from the heavily congested network appear to be longer than those from the moderately or less congested network. The general impressions regarding spatial arrangements of PASs in the network are that PASs with long total link lengths are mostly scattered at outer suburbs of the region where flows are initiated, while those with short total link lengths are predominantly located at inner suburbs and the central city where the main corridors are formed. As congestion increases, PASs with long or extremely long total link lengths tend to occur less frequently, whereas those with short total link lengths prevail throughout the network. The reason is that the routes with long or extremely long total link lengths could no longer be least cost as congestion increases. In addition, with increasing congestion there is an increasing number of PASs along the corridors; most of them are relatively short, thereby providing travelers with more alternatives to avoid congested segments of the corridors or to alleviate congestion over the corridors.

DIFFERENCES BETWEEN COMPUTED AND EXACT PROPORTIONALITY

Selected paired segments analysis at disaggregate levels in the previous section shows that there exist minimal variations in computed and exact proportionality for O-D pairs with small flows and there seem to be very perfect adherences to exact proportionality for O-D pairs with large flows. When multiplying exact proportions of flows for each O-D pair with corresponding O-D flows, O-D pairs with small flows do not seem to cause any major difference between computed and exactly proportional route flows. However, the differences may be substantial for O-D pairs with large flows and these could be consequential in applications. Therefore, investigating the magnitude of differences between the two proportional route flows is essential in determining whether the extremely small variations of computed proportionality have any significant practical implications in interpreting uniquely determined UE route flows or whether they can simply be neglected from further considerations. Recall that the computed proportionality involves the flows from only one disaggregate O-D pair traversing each segment, whereas the exact proportionality involves the flows from all disaggregate O-D pairs traversing the same segment.

Figures 5(a)-5(c) show the magnitude of differences between computed and exactly proportional route flows for the 0.20, 0.10, and 0.05 solutions respectively. Each data point represents one O-D pair. In each plot, the x-axis represents total O-D flows. The y-axis represents the absolute difference between computed and exactly proportional route flows for routes traversing a selected segment. Plots on the left and right columns correspond to the routes traversing segment 1 and 2 respectively. Those on the top row include all routes traversing a selected segment and having either positive or negative differences of flows. In order to help explore fundamental characteristics of the differences, positive and negative values are plotted separately and shown in the middle and bottom rows of each figure. Each point is plotted on log scale to enhance the visualization of extremely small differences for O-D pairs with extremely small flows. All plots are on the same scales of x and y axes for easy comparison. Absolute values on the y-axis are simply needed to enable negative differences to be displayed on log scale. Figures 5(a)-5(c) should be interpreted in conjunction with Figure 3(b).

The plots in each solution show that when computed O-D flows for routes traversing one segment exceeds the exactly proportional route flows in the positive direction, those traversing the other segment will evenly exceed the exactly proportional route flows in the negative direction. This remarkable characteristic is seen much more clearly by comparing plots side by side as shown in the middle and bottom rows of each solution. Precise similarity of the visual appearances of the patterns and their magnitude of differences are naturally governed by complementarity of total O-D flows over the two selected segments. Failure to comply with complementarity will lead to violations of proportionality in the solution, and arbitrary differences in the patterns would occur. Therefore, similarity of the differences between two proportional route flows over the same selected segments is an essentially desirable characteristic for any solution that aims to satisfy proportionality.

As seen in the plots of each figure, computed and exactly proportional route flows differ marginally in the range of very small values between -18 and -3 orders of magnitude. The observed magnitude of differences is influenced more heavily by increasing O-D flows. In light of these results, this magnitude of differences appears to be insufficiently significant to influence the entire analysis. It is of interesting to note that the notable differences between the two proportional UE route flows for O-D pairs with total flows greater than 1E-3 veh/h, which seem to correspond best to the exact proportionality as shown by the precise single horizontal alignment of data points for plots in the first two rows of Figure 3(b), reveal that there seems to be no perfect fit to the exact proportionality. This characteristic was not expected at the outset of this study.

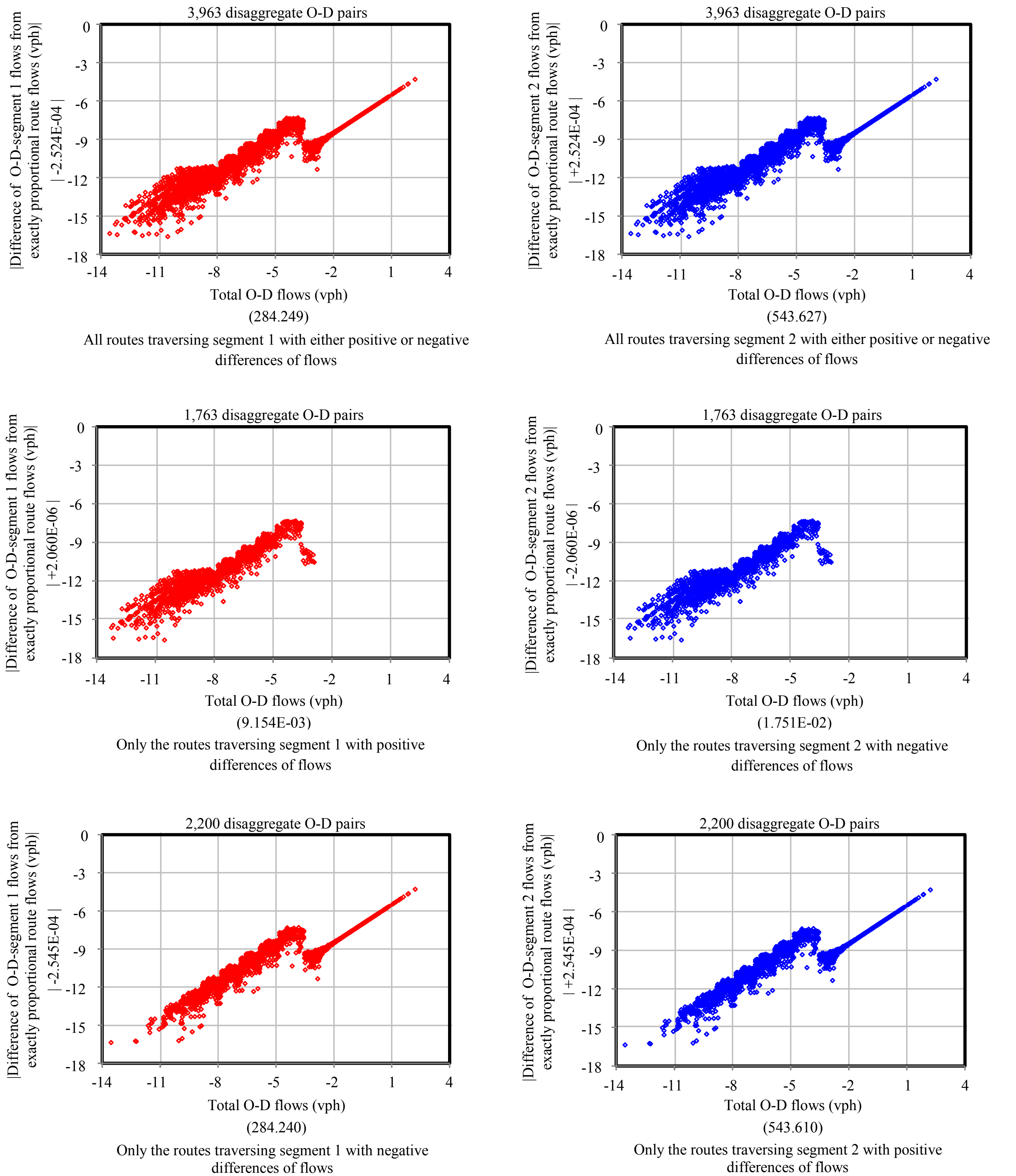


FIGURE 5(a) Differences between computed and exactly proportional route flows for the 0.20 solution.

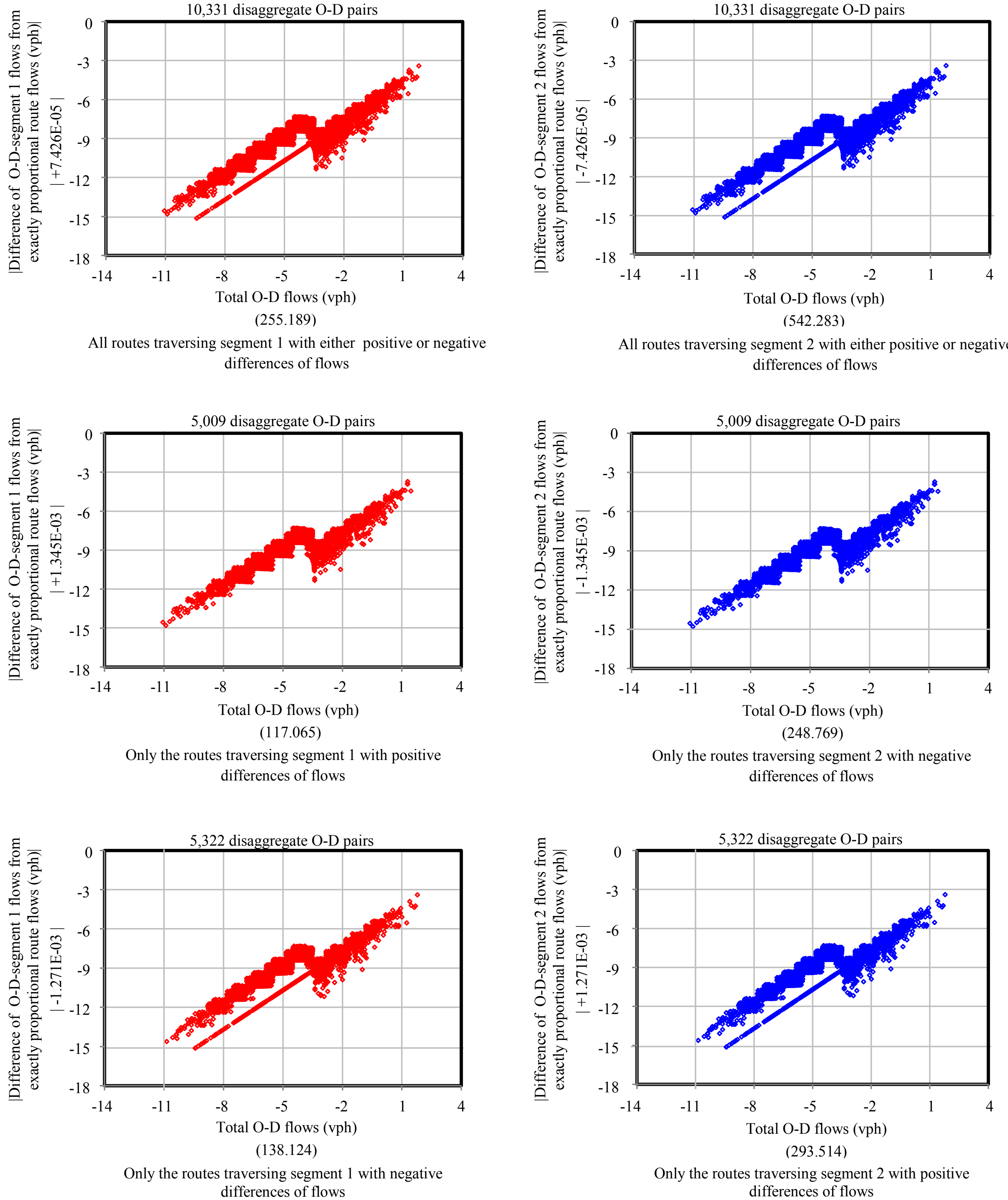
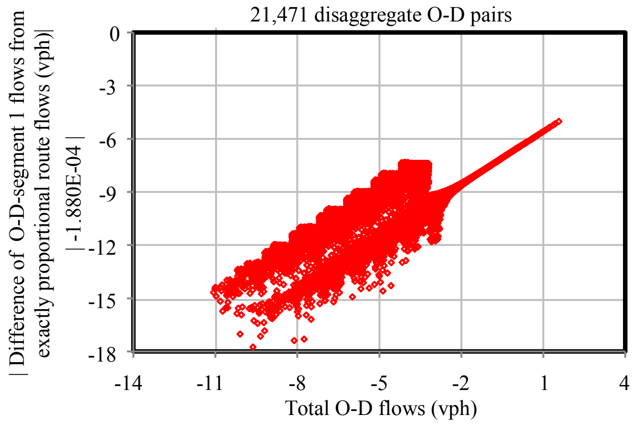
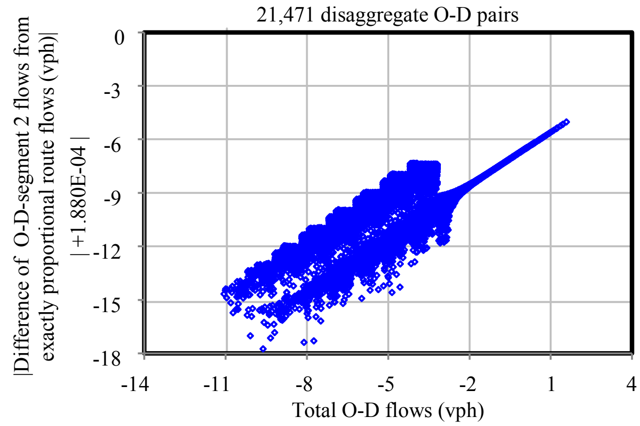


FIGURE 5(b) Differences between computed and exactly proportional route flows for the 0.10 solution.



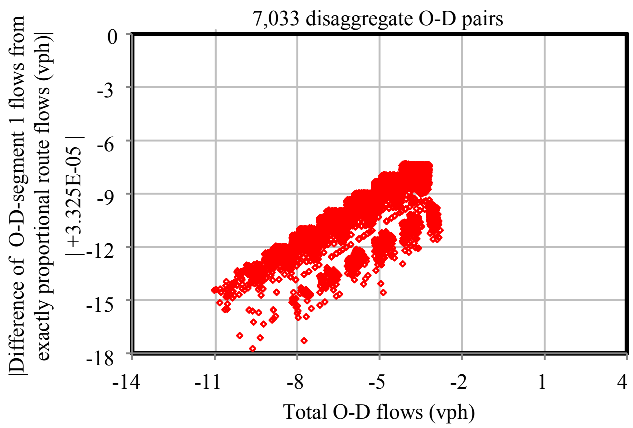
(108.502)

All routes traversing segment 1 with either positive or negative differences of flows



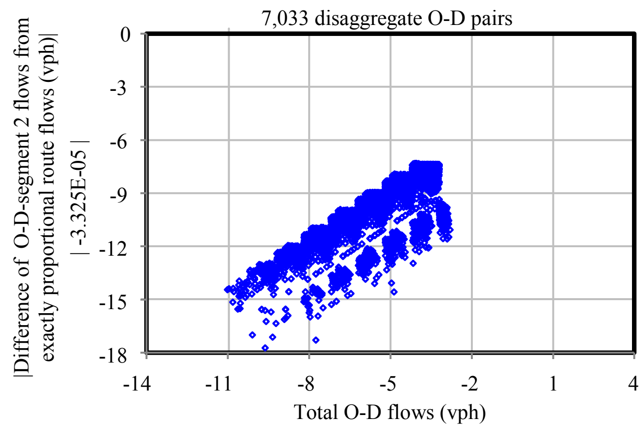
(585.433)

All routes traversing segment 2 with either positive or negative differences of flows



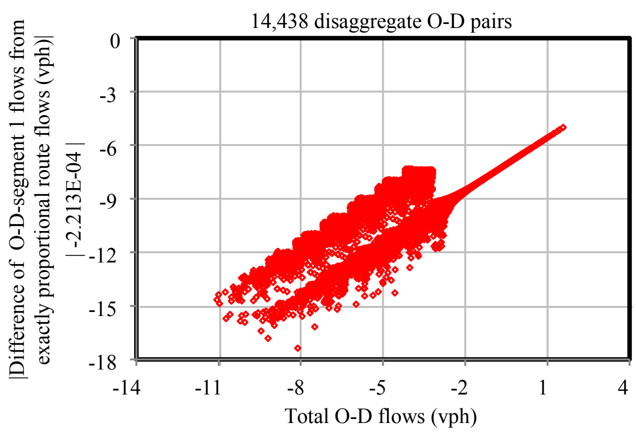
(8.870E-02)

Only the routes traversing segment 1 with positive differences of flows



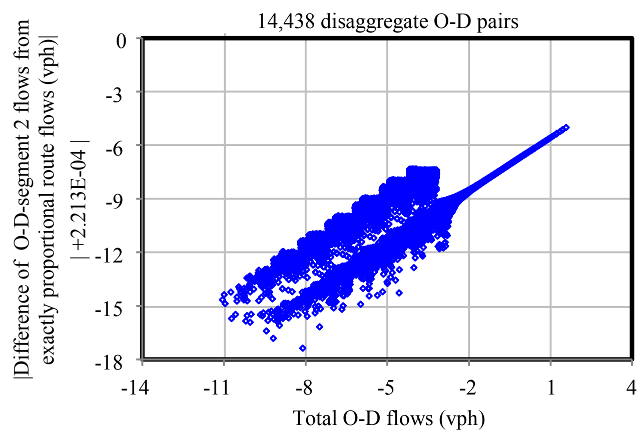
(4.788E-01)

Only the routes traversing segment 2 with negative differences of flows



(108.413)

Only the routes traversing segment 1 with negative differences of flows



(584.955)

Only the routes traversing segment 2 with positive differences of flows

FIGURE 5(c) Differences between computed and exactly proportional route flows for the 0.05 solution.

In order to determine overall significances of the magnitude of differences between the two proportional UE route flows in each solution, the number in parentheses on the x-axis label shows the total O-D flows on all the routes traversing one of the two alternative segments; the number in absolute value symbol on the y-axis label shows either net or total differences of the two proportional UE flows on all routes traversing a corresponding segment. Absolute aggregate differences of the two proportional UE route flows with respect to total O-D flows, which is the ratio of the number on the y-axis label to the number on the x-axis label, fall in the very narrow range of 1.369E-7 to 1.733E-6 for the net differences and in the somewhat wider range of 3.782E-7 to 3.749E-4 for the total differences. Such relatively small ratios appear to justify the use of uniquely determined UE route flows yielded by TAPAS in practical applications. Notice that the numbers on the x-axis labels are actually the total flows on a segment found previously in Figure 2. Since the plots in the top row are a composite of plots in the middle and bottom rows, the sum of the numbers on the x and y axis labels for plots in the middle and bottom rows is equal to the number found on the corresponding x and y axis labels for plots in the top row.

CONCLUSIONS, USEFULNESS, AND FUTURE RESEARCH

This empirical research aims to advance understandings of UE traffic assignments with uniquely determined route flows through various assessments of adherence to the condition of proportionality, which are performed for one selected pair of alternative segments over three congestion scenarios. The results show that route flows over the two segments determined by TAPAS nearly perfectly adhere to exact proportionality. Due to numerical errors, only minor differences occur between computed and exactly proportional UE route flows. Systematic solution characteristics between routes traversing each of the two alternative segments assure that TAPAS behaves properly according to the condition of proportionality. Insights from these empirical results may help transportation planning professionals to be aware of the magnitude of differences in UE route flows based on the proportionality condition, and to decide whether such differences are important for their analyses. The results may also be useful to software developers in seeking improved adherence to proportionality in route flow solutions. Essentially desirable solution characteristics of TAPAS can be used as a basis for transportation planning professionals in differentiating uniqueness from non-uniqueness of route flows in UE traffic assignment.

Since the results presented in this paper pertain only to one PAS, studies with more PASs are warranted to establish mere definite conclusions. Perhaps, a set of PASs with common merge and diverge nodes may merit future explorations. Another research direction is to determine an acceptable level of proportionality in a solution by seeking how large the relative differences between the computed and exactly proportional UE route flows should be tolerated in practical applications without changing the results of aggregate benefits. Experimenting with placing a lower bound on the O-D flows, such as 1E-4 vph, would be also useful to know whether such a bound would improve the convergence of proportionality. Assessing whether the condition of proportionality is observed in reality is a good subject for future research as well.

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